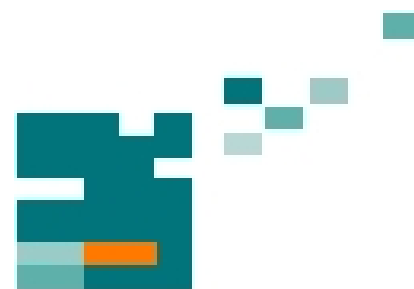


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NOISE CHARACTERISATION OF A LOW-NOISE THREE-BRANCH DIVERSITY RECEIVER AT 2.45 GHZ

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ABSTRACT

In this work, the noise characterisation of a three-branch digital receiver at 2.45 GHz is investigated. Usually, RF components are characterised by noise current and voltage sources, but here the model of noise power waves is used instead. The introduction of noise waves greatly simplifies the analysis of complex systems, because noise waves can be analysed by means of the familiar scattering parameters and signal flow graphs similar to ordinary (i.e. deterministic) signals. The model and the characterisation procedure presented are able to predict the output noise power of each channel and the mutual coupling between the channels with an uncertainty of less than 1 % as verified by measurements of several three-port antennas. An accurate noise model is essential to the realistic characterisation of multi-branch receivers for diversity or MIMO applications.

Index Terms— Noise Power Waves, Noise Characterisation, MIDIAS

1. INTRODUCTION

Modern communications systems employ multiple radio channels to benefit from diversity. This approach allows mitigation of channel fading and the suppression of unwanted signals (interference), e.g. by switching the radiation patterns of an antenna array. In order to investigate the implications of mutual radiator coupling on the performance of mobile communications systems, a three-branch digital diversity receiver was fabricated within the framework of the R&D project MIDIAS (Miniaturised Diversity Antennas for Satellite Communications) [1, 2, 3, 4]. The main objective of the project was the development and the characterisation of highly compact antenna arrays and feed networks (decoupling and matching networks) for application in small mobile devices. For the development of realistic design criteria for the interaction between a mutually coupled antenna array and the multi-port receiver, the noise parameters of the RF system must be

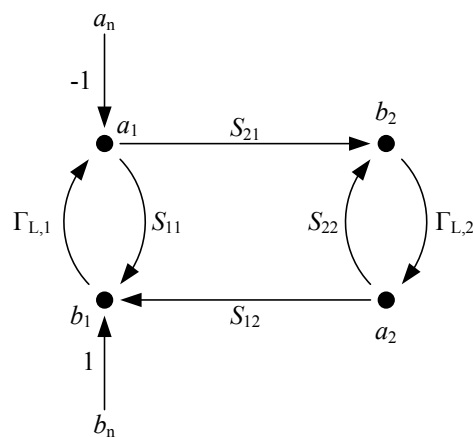


Fig. 1. Signal flow graph of a noisy two-port

known. The receiver developed in this work operates at 2.45 GHz, employs quadrature phase shift keying, and enables a real-time data processing.

2. NOISE POWER WAVES

In the theory of noise power waves the noise is modelled as additional outgoing power waves superimposed on the familiar signal waves a_i and b_i at each port of a multi-port junction. In the special case of a two-port the outgoing noise power wave at the second port can be treated as an incoming wave at the first port [5, p. 52]. The noise wave model is in fact similar to the model of noise current and voltage sources, which also allows the placement of two noise sources – one voltage and one current source – at one port of the two-port and treat the second port as noise free. The signal flow graph of a noisy two-port including the noise waves is shown in figure 1. S_{11} , S_{12} , S_{21} and S_{22} are the S parameters of the two-port. $\Gamma_{L,1}$ and $\Gamma_{L,2}$ are the load reflection coefficients at the ports 1 and 2. a_n and b_n are the noise power waves generated by the two-port. a_1 and a_2 are the signal power waves travelling into port 1 and 2, respectively and b_1 and b_2 represent the cor-

responding outgoing waves. The factors 1 and -1 are arbitrary and adopted from the notation in [5, p. 52].

In principle it is possible to convert the classical equivalent circuit diagram of the noisy two-port with a noise voltage source and a noise current source at one port into noise power waves. Thereby the voltage U_{n1} and the current I_{n1} are the equivalent noise voltage and the equivalent noise current of the noisy two-port. $Z_{0,1}$ is the reference impedance of the port where the noise sources are placed. $Z_{0,1}^*$ is the conjugate complex of the reference impedance. The incoming noise power wave a_n and the outgoing noise power wave b_n are defined by the following equations [5, p. 51].

$$a_n = \frac{U_{n1} + Z_{0,1} I_{n1}}{2\sqrt{|\operatorname{Re}\{Z_{0,1}\}|}} \quad (1)$$

$$b_n = \frac{U_{n1} - Z_{0,1}^* I_{n1}}{2\sqrt{|\operatorname{Re}\{Z_{0,1}\}|}} \quad (2)$$

An example for a noise wave is the noise power wave generated by an arbitrary load impedance. Since the noise power wave is a statistical quantity, merely the average power of the wave can be stated. This value is given by the expected value $E\{\dots\}$ of the square of the absolute value of the noise power wave. The noise power wave a_L created by this impedance is defined as follows [6, p. 60].

$$E\{|a_L|^2\} = kBT(1 - |\Gamma_L|^2) \quad (3)$$

In this case Γ_L is the reflection coefficient of the load seen from the source. k denotes the Boltzmann constant, B the bandwidth under consideration and T is the absolute temperature of the load. Complex systems with several noise power waves such as the digital multiport receiver can be handled comfortably in matrix notation. In general, a system with \vec{x} as the input vector, \vec{y} as the output vector and \tilde{A} as the transfer matrix can be written as:

$$\vec{y} = \tilde{A} \cdot \vec{x} \quad (4)$$

The input vector \vec{x} consists of the noise power waves of the load impedances at the input ports of the receiver. The output vector \vec{y} represents the noise power at each channel of the receiver after downconversion into the complex baseband. This parameter can be measured [2]. Since the noise has a Gaussian distribution with a zero expectation value, the covariance matrix of a vector comprising noise power waves provides all the required information about the power and the coupling between the channels. The covariance matrix \tilde{R}_{xx} of \vec{x} for example is

$$\tilde{R}_{xx} = E\{\vec{x} \vec{x}^H\}, \quad (5)$$

where \vec{x}^H is the Hermitian transpose (conjugate transpose) of \vec{x} . For a vector with two elements (x_1, x_2) this

covariance matrix would be:

$$\tilde{R}_{xx} = E\left\{\begin{pmatrix} |x_1|^2 & x_1 x_2^* \\ x_1^* x_2 & |x_2|^2 \end{pmatrix}\right\}; \quad (6)$$

$x_1 x_2^*$ is the covariance between the two noise power waves and the elements on the main diagonal of the matrix are the average noise power of the two waves. The noise at the output of the general system can be characterised by the covariance matrix \tilde{R}_{yy} .

$$\begin{aligned} \tilde{R}_{yy} &= E\{\vec{y} \vec{y}^H\} \\ &= E\{\tilde{A} \vec{x} [\tilde{A} \vec{x}]^H\} \\ &= E\{\tilde{A} \vec{x} \vec{x}^H \tilde{A}^H\} \\ &= \tilde{A} \cdot E\{\vec{x} \vec{x}^H\} \cdot \tilde{A}^H \\ &= \tilde{A} \cdot \tilde{R}_{xx} \cdot \tilde{A}^H \end{aligned} \quad (7)$$

With that it is possible to calculate the output noise power waves if the input noise and the transfer matrix \tilde{A} of the system are known.

3. NOISE CHARACTERISATION OF A SINGLE RECEIVER CHANNEL

It is possible to measure the noise power and the correlation between the three channels of the digital receiver after the demodulation. The values that are displayed by the appropriate software are not the real power with the correct physical dimension but they are proportional to the real values [2]. However, this information is sufficient for the noise model of the receiver. The single receiver channel can be described by two parameters. S_{11} is the reflection coefficient of the input port and G is the transmission coefficient of the branch from the applied RF signal to the complex baseband representation in the receiver software. The corresponding noisy signal flow graph is shown in figure 2. Γ_L is the reflection coefficient of the load at the input port and b_2 is the noise power wave at the output of the receiver channel. The displayed value of the noise power is proportional to $E\{|b_2|^2\}$. a_n , b_n and a_L are the noise power waves generated by the receiver channel and by the load.

The bandwidth of the receiving system is 1 MHz. Within this bandwidth the received noise power density can be assumed constant. As the measured value is proportional to the real power, the bandwidth B is chosen as 1 Hz in the following. The effect is that the value of the transmission coefficient is scaled by 10^6 . The noise power wave b_2 is given by [7, 8]:

$$b_2 = a_L C - a_n C + b_n D \quad (8)$$

with:

$$\begin{aligned} C &= \frac{G}{1 - S_{11} \Gamma_L} \\ D &= \frac{\Gamma_L G}{1 - S_{11} \Gamma_L}. \end{aligned} \quad (9)$$

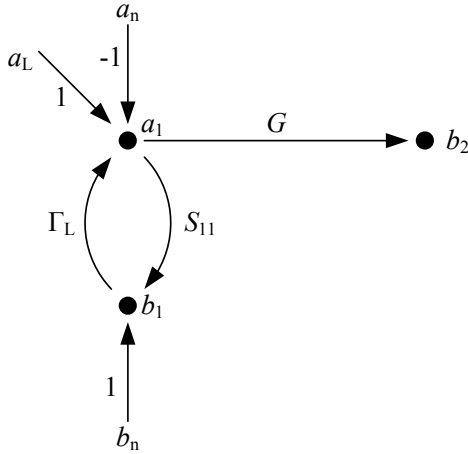


Fig. 2. Signal flow graph of a single noisy receiver channel

To calculate the measurable parameter $E\{|b_2|^2\}$, (8) has to be written in matrix notation as in (4).

$$b_2 = \vec{A} \cdot \vec{n} = \begin{pmatrix} C & -C & D \end{pmatrix} \cdot \begin{pmatrix} a_L \\ a_n \\ b_n \end{pmatrix} \quad (10)$$

$E\{|b_2|^2\}$ can then be calculated according to (7):

$$\begin{aligned} \tilde{R}_{b_2 b_2} &= E\{b_2 b_2^H\} \\ &= \vec{A} \cdot E \left\{ \begin{pmatrix} |a_L|^2 & 0 & 0 \\ 0 & |a_n|^2 & a_n b_n^* \\ 0 & a_n^* b_n & |b_n|^2 \end{pmatrix} \right\} \cdot \vec{A}^H \end{aligned} \quad (11)$$

The covariance between the noise power wave a_L from the load and the noise power waves a_n and b_n equals zero. They are uncorrelated because they are generated in different components of the measurement setup: a_L is generated in the load and a_n and b_n are generated in the receiver.

The transmission coefficient G can be calculated from the measurement of the output noise power with two different known noise temperatures of the load [6, p. 362]. Therefore, a standardised noise source is connected to the input port of the receiver channel.

$$G = \frac{P_h - P_0}{(T_h - T_0) k B} \quad (12)$$

The output noise power is measured two times. First with the cold noise temperature at T_0 and then with the hot noise temperature T_h . The used noise source has an excess noise ratio of 14.7 dB. P_0 is the measured output noise power with the cold noise source at the input, P_h is the noise power at the output of the channel with the hot noise source. With that result there are solely four parameters in equation (11) that are unknown: $|a_n|^2$, $|b_n|^2$ and the complex value of $a_n b_n^*$. To calculate them, the output noise power is measured for

four different loads at the input port. The reflection coefficient of the load influences the parameters C and D so that an equation system with four equations and four unknowns can be set up. By numerical solution, the noise parameters $|a_n|^2$, $|b_n|^2$ and $a_n b_n^*$ can be determined. The four reference impedances that were used are:

$$\begin{aligned} Z_1 &= (0.6510 + j23.815) \Omega \\ Z_2 &= (183.30 - j121.05) \Omega \\ Z_3 &= (38.755 - j2.5512) \Omega \\ Z_4 &= (1.4356 - j65.240) \Omega \end{aligned}$$

The measured power values are chosen as 10^{-15} W. This factor is arbitrary and produces a transfer factor G that is in a realistic range. The measured noise power with the cold and hot noise source and the calculated transfer and noise factor is presented in table 1. The noise factor F is calculated by using the Y factor method [6, p. 127].

$$Y = \frac{P_h}{P_0} \quad (13)$$

$$F = \frac{T_h/T_0 - 1}{Y - 1} \quad (14)$$

P_0	76.20
P_h	1447
G	70.64 dB
F	2.15 dB

Table 1. Measured relative noise power at the output of the first receiver channel and the appropriate transmission and noise factor

The measured noise power for the four different reference load impedances at the input port of the first channel are shown in table 2.

$R_{b_2 b_2 1}$	33.10
$R_{b_2 b_2 2}$	72.68
$R_{b_2 b_2 3}$	73.14
$R_{b_2 b_2 4}$	44.49

Table 2. Measured relative noise power at the first receiver channel with the four different reference loads at the input of the receiver

With these measurement results, the noise parameters of the channel can be calculated. The measurement procedure has to be performed for all channels of the receiver. This is the basis for the overall noise characterisation of the three-branch receiver. The measurement setup is shown in figure 3. The calculated noise power waves in table 3 are valid for the renormalised scattering parameters of the channel. That means the S parameters of the channel are normalised to the conjugate

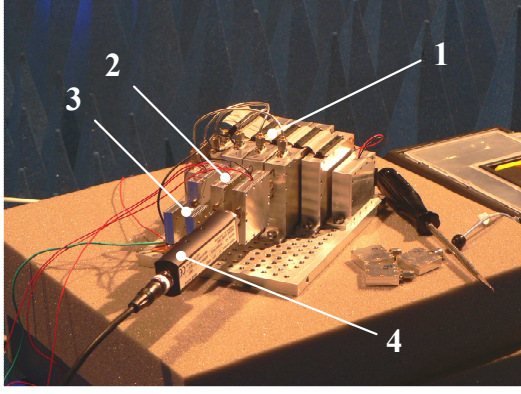


Fig. 3. Digital receiver (1) with preamplifiers(2), reference loads (3) and noise source (4)

complex of its own input impedance to generate an S_{11} that equals zero [9, p. 222]. This is an important step to simplify the signal flow graph of the whole receiver. The complexity of the signal flow graph is pushed now into the renormalisation of the S parameters.

$E\{ a_n ^2\}$	$2.5328 \cdot 10^{-21} \text{ W}$
$E\{ b_n ^2\}$	$0.6908 \cdot 10^{-21} \text{ W}$
$E\{a_n b_n^*\}$	$(-0.2331 + j0.2295) \cdot 10^{-21} \text{ W}$

Table 3. Noise power waves and their covariance of the first receiver channel

With the knowledge of the noise parameters of the receiver channel it is possible to calculate an expected noise power at the output of the branch for an arbitrary load impedance. Some examples for the comparison of the calculated and the measured relative noise powers are given in table 4. The uncertainty of the calculated results is better than 1 % in every case.

$Z [\Omega]$	$R_{b_2 b_2, \text{meas}}$	$R_{b_2 b_2, \text{calc}}$
$14.7 - j9.23$	57.1	57.3
$0.48 + j17.4$	32.9	32.7
$430 - j120$	63.5	63.9
$50.0 + j0.00$	76.0	76.2

Table 4. Comparison of calculated and measured relative noise powers at the output of the first receiver channel for several input load impedances

4. NOISE CHARACTERISATION OF THE THREE-BRANCH RECEIVER

The digital receiver uses three branches. Usually there will be a three-port antenna connected to the receiver ports, which will generate mutual coupling between the three channels. To calculate the noise power and the coupling between the channels, the noise parameters of

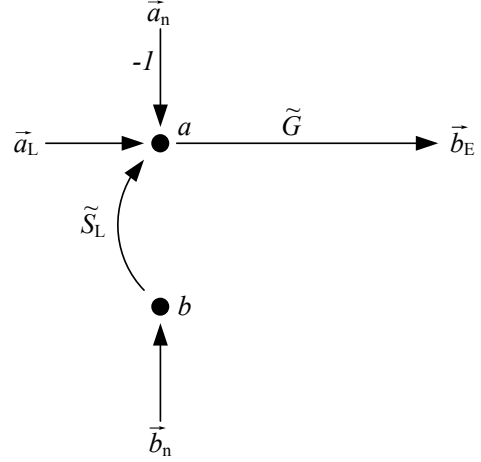


Fig. 4. Matrix signal flow graph of the multi port receiver with renormalised ports

each channel have to be known. For the characterisation of the receiver, the signal flow graph must be set up. This graph is very complicated because it includes eight loops that must be accounted for in the calculations. To reduce the number of loops, it is possible to eliminate the reflection coefficients of the receiver ports by renormalising them to their own conjugate complex input impedance. If every port of the receiver is renormalised, the signal flow graph is free of any loops. That makes it easy to use a matrix signal flow graph of the receiver as shown in figure 4 [10]. The measured S parameters of the first receiver channel that are normalised to 50Ω are:

$$\tilde{S}_{E.1} = \begin{pmatrix} 0.213 + j0.116 & 0 \\ 3406.01 & 0 \end{pmatrix} \quad (15)$$

By renormalising this S matrix at port one to the conjugate complex of the input impedance of this port it changes to:

$$\begin{aligned} \tilde{S}_{E.1, \text{norm}} &= \begin{pmatrix} 0 & 0 \\ 3473.31 + j511.95 & 0 \end{pmatrix} \\ &= \begin{pmatrix} 0 & 0 \\ G_{E.1} & 0 \end{pmatrix} \end{aligned} \quad (16)$$

The new input reflection coefficient is zero and the transmission coefficient has a complex value now.

The S parameters of the network that is connected to the receiver have to be renormalised, too. Hence the normalising impedances are complex the normalising impedance of the load has to be the conjugate complex of the normalising impedance of the port to which the load is connected to. In other words, the normalising impedances of the load terminals are equal to the input impedances of the receiver ports.

The vectors \vec{a}_L , \vec{a}_n and \vec{b}_n contain the noise waves of the load and the receiver channels. \tilde{S}_L is the scattering matrix of the load network that is renormalised

to the appropriate ports of the receiver. \tilde{G} is the matrix that comprises the transmission factor of every single receiver channel on its main diagonal.

$$\tilde{G} = \begin{pmatrix} G_{E.1} & 0 & \dots & 0 \\ 0 & G_{E.2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & G_{E.N} \end{pmatrix} \quad (17)$$

If every port is correctly normalised, this signal flow graph is valid for a receiver with an arbitrary number of ports. The noise power waves at the output of the receiver are combined in the vector \vec{b}_E . This vector is calculated as:

$$\vec{b}_E = \tilde{G} \vec{a}_L - \tilde{G} \vec{a}_n + \tilde{G} \tilde{S}_L \vec{b}_n \quad (18)$$

To calculate the noise behaviour, this equation has to be written as a general system as in (4):

$$\begin{aligned} \vec{b}_E &= \begin{pmatrix} \tilde{G} & -\tilde{G} & \tilde{G} \tilde{S}_L \end{pmatrix} \cdot \begin{pmatrix} \vec{a}_L \\ \vec{a}_n \\ \vec{b}_n \end{pmatrix} \\ &= \tilde{A} \cdot \vec{n} \end{aligned} \quad (19)$$

The noise power and the covariance between the noise of the different receiver channels are combined by the covariance matrix $\tilde{R}_{b_E b_E}$ of \vec{b}_E that is defined as:

$$\tilde{R}_{b_E b_E} = \tilde{A} \tilde{R}_{nn} \tilde{A}^H \quad (20)$$

\tilde{R}_{nn} is the covariance matrix of the input noise power waves and can be written as:

$$\tilde{R}_{nn} = E\{\vec{n} \vec{n}^H\} = \begin{pmatrix} \tilde{A}_L & \tilde{0} & \tilde{0} \\ \tilde{0} & \tilde{A}_n & \tilde{\rho}_{cov} \\ \tilde{0} & \tilde{\rho}_{cov}^* & \tilde{B}_n \end{pmatrix} \quad (21)$$

with:

$$\begin{aligned} \tilde{A}_n &= E \left\{ \begin{pmatrix} |a_{n1}|^2 & 0 & \dots & 0 \\ 0 & |a_{n2}|^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & |a_{nN}|^2 \end{pmatrix} \right\} \\ \tilde{B}_n &= E \left\{ \begin{pmatrix} |b_{n1}|^2 & 0 & \dots & 0 \\ 0 & |b_{n2}|^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & |b_{nN}|^2 \end{pmatrix} \right\} \\ \tilde{\rho}_{cov} &= E \left\{ \begin{pmatrix} a_{n1} b_{n1}^* & 0 & \dots & 0 \\ 0 & a_{n2} b_{n2}^* & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & a_{nN} b_{nN}^* \end{pmatrix} \right\} \end{aligned}$$

The noise power generated by a multi-port load network is defined as [6, p. 60]:

$$\tilde{A}_L = E\{\vec{a}_L \vec{a}_L^H\} = k B T (\tilde{I} - \tilde{S}_L \tilde{S}_L^H) \quad (22)$$

As the transmission factor of each channel is merely known by its absolute value, the results are solely correct in their absolute values. That does not affect the result for the noise power and the absolute value of the covariance but the angle of the complex covariance. To get to know the correct value of the complex covariance it is necessary to know the phase of the transmission factor. Therefore, a cable with constant phase was measured with a vector network analyzer and subsequently connected between the first and the second port of the receiver. In the noise model, the phase of the first channel was set to zero and the phase of the second branch was set to a value that produces the same calculated angle of the covariance of the output noise power as measured. This procedure was repeated for the third channel so that the angles of the transmission coefficient of all channels were determined.

The noise model of the receiver was verified by several measurements. For example a three-port T-junction fabricated by soldering three semi-rigid coaxial lines together was measured. The comparison of the measurement of the noise and the calculation shows a very good accuracy of the model. The measurement of \tilde{R}_{meas} gives:

$$\begin{pmatrix} 43.1 & 15.70 \angle -58^\circ & 5.8 \angle +92^\circ \\ 15.7 \angle +58^\circ & 68.6 & 19.5 \angle +149^\circ \\ 5.8 \angle -92^\circ & 19.5 \angle -149^\circ & 52.5 \end{pmatrix}$$

The calculation of the noise by the model gives \tilde{R}_{calc} as:

$$\begin{pmatrix} 43.5 & 15.9 \angle -58.8^\circ & 6.0 \angle +89.2^\circ \\ 15.9 \angle +58.8^\circ & 68.6 & 19.2 \angle +149.4^\circ \\ 6.0 \angle -89.2^\circ & 19.2 \angle -149.4^\circ & 52.4 \end{pmatrix}$$

These noise power values are given on the arbitrary scale of 10^{-15} W.

As a second example of a noise measurement, a three-port antenna was used. The measurement of this compact array of three $\lambda/4$ monopoles was done in an anechoic chamber to avoid unwanted signals during the noise measurement. This also guarantees an antenna temperature of 290 K. The spacing between the elements is $\lambda/10$ and the antenna does not use a matching and decoupling network. The measurement setup of the receiver with this antenna is shown in figure 5. The measurement of \tilde{R}_{meas} gives:

$$\begin{pmatrix} 73.6 & 12.1 \angle +27^\circ & 4.2 \angle -79^\circ \\ 12.1 \angle -27^\circ & 37.9 & 9.7 \angle -22^\circ \\ 4.2 \angle +79^\circ & 9.7 \angle +22^\circ & 64.8 \end{pmatrix}$$

The calculation of the noise by the model gives \tilde{R}_{calc} :

$$\begin{pmatrix} 72.2 & 11.3 \angle +24.5^\circ & 3.3 \angle -95.1^\circ \\ 11.3 \angle -24.5^\circ & 37.6 & 8.7 \angle -20.2^\circ \\ 3.3 \angle +95.1^\circ & 8.7 \angle +20.2^\circ & 63.5 \end{pmatrix}$$

These noise power values are given on the arbitrary scale of 10^{-15} W as defined before.

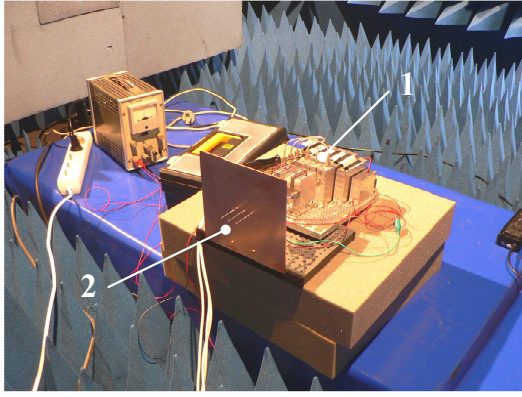


Fig. 5. Measurement setup for the noise measurement of the receiver (1) with $\lambda/4$ monopole array antenna (2)

5. CONCLUSION

The noise characterisation of a three-branch diversity receiver by the use of noise power waves was presented. The developed noise model of the receiver can predict the noise power at the output of the receiver with an uncertainty of less than 1 %. The validity of the model was verified by several measurements with radiating and non radiating load networks. In principle the model is valid for an arbitrary number of receiver channels. The techniques outlined in this paper facilitate the characterisation of mutually coupled diversity systems in terms of the received SNR [2].

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